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Abstracts

Chemical Indices of Graphs with Degree Sequences

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The chemical indices, such as the Wiener index, ABC index, Harry index, Zagreb Index etc, of a graph have received a lot of attention. In this talk, we introduce some progress and new results on these chemical indices of graphs with given degree sequences. In addition, some problems are concluded.

Simplicial complexes of a polyomino tilings

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A polyomino is a plane geometric figure formed by joining one or more equal squares edge to edge. Polyomino tiling problem asks if it is possible to cover given subset R of regular square grid using given set S of polyomino shapes. This problem is also studied in context of regular square grids on surfaces. It is also of interest in combinatorics to find the total number of regular polyomino tilings.

In this lecture we introduce a simplicial complex related to polyomino tiling problems. Namely, a i-simplex of this complex corresponds to a placement of some i polyomino shapes onto R. It turns out that combinatorics of this simplicial complex keeps a lot of information about the polyomino tiling in consideration. Thus, it is interesting to study its topological and algebraic properties of its Stainley-Reisner ring. Some nice combinatorial identities are proved appealing on the Euler characteristic of complex. Using discrete Morse theory we prove that in some cases this complexes have the homotopy type of a wedge of spheres.

Graph-indexed random walks

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I will survey my results regarding graph-indexed random walks. A graph-indexed random walk on graph G is a mapping $f : V(G) \to \mathbb{Z}$ such that there exists one vertex $r \in V(G)$, let us call it the root, with $f(r) = 0$ and such that for every $uv \in E(G)$, it holds $|f(u) - f(v)| \leq 1$.

Graph-indexed random walks are sometimes called Lipschitz mappings of graphs. Furthermore, the average range of a rooted undirected graph is the arithmetic average of the sizes of homomorphic images of all possible Lipschitz mappings of the graph. I will show results on average range of graphs. Namely, the precise formulas for various classes, a recent progress in attacking the Loebl-Neetil-Reed conjecture on average range, and, finally, some algorithmic results regarding the parameter. I will conclude with open problems.

Part of the work is a joint research with Jaroslav Nešetřil.

Keywords: random walks, graphs, algorithms, homomorphisms, Lipschitz mappings

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Simultaneous representation problem for interval and circular-arc graphs in non-sunflower position

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Lubiw and Jampani introduced the *simultaneous representation problem* in 2009. The simultaneous representation problem for a given class of intersection graphs asks if some k graphs can be represented so that every vertex is represented by the same object in each representation. We say that k graphs are in sunflower position if the intersection of any two of them is the same.

We will summarize known results in the area, then we prove that it is NP-complete to decide the simultaneous representation problem for the class of interval and circular-arc graphs in the case when k is a part of the input and graphs are not in a sunflower position.

Keywords: intersection graphs, simultaneous problem, NP-completeness

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Some results and problems on unique-maximum colorings of plane graphs

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IA unique-maximum coloring of a plane graph G is a proper vertex coloring by natural numbers such that each face α of G satisfies the propety: the maximal color that appears on α, appears precisely on one vertex of α (or shortly, the maximal color on every face is unique on that face). Fabrici and Göring proved that six colors are enough for any plane graph and conjectured that four colors suffice. Thus, this conjecture is a strengthening of the Four Color Theorem. Wendland later decreased the upper bound from six to five.

We first show that the conjecture holds for various subclasses of planar graphs but then we disprove it for planar graphs in general. Thus, the facial unique-maximum chromatic number of the sphere is not four but five. In the second part of the talk, we will consider various new directions and open problems.

(Joint work with Vesna Andova, Bernard Lidický, Borut Lužar, and Kacy Messerschmidt)

Keywords: graph, planar graph, coloring, Four Color Theorem

The cycle index of the automorphism group of $\mathbb{Z}_n$

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We consider the group action of the automorphism group $\mathcal{U}_n = \mathrm{Aut}(\mathbb{Z}_n)$ on the set $\mathbb{Z}_n$, that is the set of residue classes modulo n. Clearly, this group action provides a representation of $\mathcal{U}_n$ as a permutation group acting on n points. One problem to be solved regarding this group action is to find its cycle index. Once it is found, there appears a vast class of related enumerative and computational problems with interesting applications. We provide the cycle index of specified group action in two ways. One of them is more abstract and hence compact, while another one is basically procedure of composing the cycle index from some *building blocks*. However, those *building blocks* are also well explained and finally presented in very detailed fashion.

Keywords: cycle index, automorphism group

An Axiomatic Approach to Summation

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Everybody seems to know what summation in mathematics is all about. But not so fast! There is a beautiful axiomatic approach to summation as a cumulation process that deserves a careful mathematical investigation. Our interest is to discuss finitary and arbitrary summation structures in connection with socalled (finitary) summoids. The latter correspond to semirings and complete semirings. An application will be an algebraic modeling of flow aggregation on networks via convolution.

Graph Coloring as a Tool for Optimization of Communication Networks Design

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Assigning operational frequency channels to communication networks while avoiding interference is similar to the vertices graph coloring, which consists in finding the minimum number of colors (span) necessary to color all vertices of the graph $G = (V, E)$, such that adjacent vertices (network transmitters) $v \in V$ have different colors from the available subset $D_v \subseteq D$, where D is the set of all the channels available for the whole network. Formally, the problem is described as:

For undirected graph $G = (V, E)$, $\{v, v\} \in E, \forall v \in V$, *sets* $T_{vw} \subset Z$, $\{v, w\} \in E$, $0 \in T_{vw}$ *demand* $c_v \in Z^+$ *domain subsets* $D_v \subseteq Z^+ \quad \forall v \in V$, $D = \cup_{v \in V} D_v$ *and positive number* K,

find and assignment subsets $f : V \mapsto 2^D$ *such that:*

1. $|f(v)| = c_v$,
2. $f(v) \subseteq D_v$,
3. $|\bar{f} - \bar{g}| \notin T_{vw}$, $\forall \{v, w\} \in E$, $\bar{f} \in f(v), \bar{g} \in f(w)$, $v \neq w$ *or* $\bar{f} \neq \bar{g}$ *and*
4. $\max \cup_{v \in V} f(v) - \min \cup_{v \in V} f(v) \leq K$.

Graph coloring is a NP-complete problem, i.e. it cannot be solved in polynomial time. Therefore, heuristic methods must be applied to solve it. Generally, these optimization techniques provide suboptimal solutions, which are perfectly acceptable for problems of large dimension. Their performance is carried out by comparing the results provided by a specific algorithm in a benchmark example versus the known optimal (the best suboptimal) solution, which, in turn, is obtained by applying mathematical methodleading to a bound for the optimization target. Sequential (greedy) algorithms are among the most popular due to their relative simplicity. Nevertheless, local search techniques, including adaptive modifications of random walk, simulated annealing, genetic and tabu search algorithms sometimes occur to be preferable.

Some enumerations in class of unicyclic graphs

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We enumerate labeled unicyclic graphs with fixed degree sequence and give recursive formula for it. Besides, we investigate the number of non-isomorphic unicyclic graphs and give a formula of its number in terms of the numbers of non-isomorphic trees.

Keywords: unicyclic graphs, labeled graphs, degree sequence.

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